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ORIGINAL ARTICLE

Examining mathematics mentor teachers' practices in professional development courses on teaching data analysis: implications for mentor teachers' programs

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Abstract In this paper, we report on the training practices of 12 mathematics mentor teachers who developed and implemented five short professional development courses after participating in a 5-month continuous professional development course "Competence-oriented teaching and learning of data analysis." The intention of this course was to deepen their professional knowledge of teaching statistics using digital tools, and to develop their competences and knowledge for developing and implementing their own professional development courses in statistics. Here, we explore how the professional program itself is reflected in the short courses they designed and implemented. Although the sample is very small, the cases allow for interesting insights into their training practices and challenges that seem to have a major impact on the quality of their professional development courses. On this basis we offer suggestions for how continuous professional development courses for mathematics mentor teachers might be designed to support their diverse needs in the professional development system.

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1 Introduction

Professional development initiatives have become a necessity in today's educational system as one driving force to support reform of school mathematics (Borko et al. 2011; Garet et al. 2001; Lipowsky 2004). Many countries (e.g., Great Britain, the Netherlands, Switzerland) have set their focus on improving education, and are investing vast resources in teacher continuous professional development (CPD). Germany is not an exception to this phenomenon. Nowadays, the German educational system is faced with many challenges, such as considerable shortage of specialized secondary subject teachers, high quota of at-risk students, rising quota of out-of-field teachers, and a high variability between different federal states (Kramer and Lange 2014). For these reasons, a national CPD institution, the German Center for Mathematics Teacher Education (DZLM), was founded in 2010 as a cooperation of seven German universities.

The objectives of the center are to promote CPD, set nationwide standards for CPD, and develop quality, needs-based CPD based on the research literature on effective CPD design principles (e.g., Darling-Hammond et al. 2009; Desimone 2011; Garet et al. 2001; Guskey 2002; Lipowsky 2004, 2011). A particular focus of DZLM is on setting up courses for mentor teachers who can "multiply" what they have learned and set up their own professional development courses for teachers. Whom can we consider to be a "mentor teacher"? In this paper, the phrase will be used as a general term for those teachers who have a role in teacher education outside universities as part of their job, in addition to teaching in schools. In Germany, for instance, mathematics mentor teachers[1] have different roles depending on the fed-

[1] From this point on we just use the term mentor instead of mathematics mentor teacher.

eral state; these may include mathematics coaches, expert advisors, and regional PD[2] organizers. They are central for providing opportunities for teachers' professional development. However, most take on one of these roles without any further professional preparation; courses for mentors with an emphasis on subject-matter-specific aspects are practically non-existent (Kramer and Lange 2014), and preparatory courses often only touch general principles for educating teachers.

Elliot et al. (2009) contend that this area has been underdefined and under-studied—we know very little about what CPD leaders need to know and be able to do—but is slowly growing in its importance. DZLM is highly interested in focusing its research on the question of how courses for mentors should be designed in order for them to improve their own training practices so they can offer more effective high quality professional development courses. To this end, DZLM develops its courses on a design-based research principle. Therefore, evaluations and retrospective analyses of DZLM courses with the aim of improving them is a basic requirement.

In this paper we try to fill in this knowledge gap—how best to design courses for mentors—by focusing on mentors' practices in designing effective and high-quality PD after participating in a five-month long CPD[3] on teaching statistics with digital tools at middle school level. This CPD aimed (1) at improving participants' content and pedagogical content knowledge for teaching statistics with digital tools and (2) at improving their PD training competence with a focus on designing effective PD courses on teaching statistics with digital tools (see Sects. 2, 3) (Biehler et al. 2013a, b; Kuzle and Biehler 2014; Kuzle et al. 2013). While we report on the former perspective in Biehler et al. (submitted), we focus here on the latter. There is a small overlap of these two papers (without literal quoting), as far as the design of the CPD course is concerned. On the basis of retrospective analysis of the mentors' practices, we indicate what consequences these have for redesigning our CPD.

In what follows, we first review literature that informed our design of the CPD course, before describing the course itself. Next, we provide a description of DZLM design principles and formulate concrete research questions for each design principle. After a section on methods, we report on our findings, which focus on the training practices that the mentors developed with emphasis on outlined designing principles. We conclude the paper by considering which factors may account for the variability of implementations and for a limited or missing implementation of the design principles.

2 Program for supporting mathematics mentor teachers' professional development: the context of the study

2.1 Guiding principles for supporting mathematics mentor teachers' professional development

Given the under-represented research base in the area of supporting mentors' professional development (Elliot et al. 2009), and no validated theory of teacher learning to inform professional development models (Borko et al. 2011), we drew from the literature on teacher professional knowledge and on designing effective CPD when designing our own CPD for mentors.

2.1.1 Professional knowledge for teaching statistics

The topic of statistics is known for being neglected in education—(preservice) teachers lack quality statistics education and inservice teachers often teach statistics without special preparation (Burrill and Biehler 2011). This was also true for our mentors. Although the "new" German national standards for mathematics (KMK 2003) require the teaching of statistics with digital tools at all school levels, our mentors had practically no experience in doing this, not even with spreadsheets, which are obligatory for use in schools. We based our course on research in statistics education on the question of what knowledge and competence is needed by teachers (Batanero et al. 2011). The goals of the professional development program were:

- developing basic knowledge and competence in doing statistics with digital tools using Fathom (content knowledge). Here we built on extensive teaching material with Fathom (Biehler et al. 2011).
- deepening pedagogical content knowledge and pedagogical technological content knowledge for teaching statistics in grades 5–10 with digital tools (here Fathom) (Biehler et al. submitted; Wassong and Biehler 2010). This model is an adaptation of well-known models of mathematical knowledge for teaching (Ball et al. 2008; Shulman 1986), and of technological knowledge (Koehler and Mishra 2005; Niess 2005). It includes four facets: (1) common and practice-oriented content knowledge, (2) content and pedagogical knowledge of curriculum, (3) pedagogical knowledge of teaching and learning, and teaching experience, and (4) common and pedagogical technological content knowledge.

[2] As opposed to CPD, we use the term PD to denote professionalization lasting no more than 1 day.

[3] The CPD "Competence-oriented teaching and learning of data analysis" for mathematics mentor teachers was designed and implemented by the authors of the paper together with Janina Oesterhaus and Thomas Wassong from the University of Paderborn.

Thus, we treated our mentors in the first place as "ordinary teachers" where we focused on topics that were identified at both the national and international level as important for modern statistics teaching with digital tools (e.g., Biehler et al. 2013a, submitted; Burrill and Biehler 2011; Garfield et al. 2012) as well as the new German national standards for mathematics (KMK 2003).

2.1.2 Professional knowledge for designing effective professional development courses

Mentors are often unprepared to take this role upon themselves (Kramer and Lange 2014), and our mentors were no exception. Therefore, the second goal of our professional development program was to systematically prepare them for their role as experts in adult education in general, and in the area of designing courses for teaching data analysis with digital tools in particular. This included developing professional knowledge for teacher training[4] (DZLM 2013), and fostering and supporting the reorganization of their existing training practices.

We distinguished between the following layers of knowledge:

- basic specificities of adult learning (Loucks-Horsley et al. 2003).
- core design principles identified across theoretical frameworks and research projects as crucial for effective CPD, such as content focus, (long-term) active learning, quality of the activities, coherence, and collaboration (e.g. Darling-Hammond et al. 2009; Desimone 2011; Garet et al. 2001; Lipowsky 2004, 2011; Loucks-Horsley et al. 2003; Putnam and Borko 2000). For more details see Rösken-Winter et al. (2015) and Jackson et al. (2015).
- DZLM's design principles with a special focus on mathematics (DZLM 2013) that serve as a foundation for every DZLM CPD, such as our course (see Rösken-Winter et al. 2015) and the mentors' own PDs. These principles were derived from theoretical models and empirical findings as outlined in the previous bullet.

2.1.3 Knowledge about models of teachers' professional knowledge for teaching statistics with digital tools

Whereas the knowledge described in Sect. 2.1.2 is primarily concerned with PD design and training principles, we based our course, in addition, on the theoretical assumption that mentors themselves need to have a model of teachers' professional knowledge in statistics (Wassong and Biehler 2010) in mind when designing any PD (see Sect. 2.1.1). This would enable them to more consciously select the content (the knowledge dimension), to explain it and organize adequate learning activities including an anticipation of possible learning difficulties (the pedagogical content dimension), and to decide how and when to use technology (the technological dimension) for their PD course on the basis of our CPD for them or from their own experience. For more details see Biehler et al. (submitted).

2.2 The specific approach to the professional development program: a longitudinal model of mathematics mentor teachers' growth

The current study took place in the context of a DZLM professional development program "Competence-oriented mathematics teaching" for secondary school mathematics mentor teachers, which was carried out in a blended-learning scenario. The program lasted an entire school year (ca. 185 working hours for the participants) and comprised two half-year modules: competence-oriented statistics teaching, and competence-oriented teaching and learning in mathematics in general. Here we focus on the first module, which focused on combining content- and process-related competences in statistics. In what follows, we describe how we structured our professional development program based on the three theoretical assumptions and knowledge sources described in Sect. 2.1.

We structured the learning sequence according to an adaptation of a three-phase model—learning-off-job, learning-by-job, and learning-on-job—as suggested by Müller (2003) and Wahl et al. (1991). This is a rather general model, which is used in professional training programs when working with novice mentors, such as PRIMAS (Maaß and Doorman 2013). The model itself is based on principles for effective professional development, embedded in a so-called "sandwich principle" with three different types of learning phase as indicated by Maaß and Doorman (2013):

> *Learning-off-job* (phase I) consists of active learning of important and fundamental knowledge instances for designing PD courses.
>
> *Learning-by-job* (phase II) consists of planning and implementing a PD, followed by reflection. This is done on the basis of knowledge acquired in the first phase and through an expert support-system (e.g., supervision, counseling).
>
> *Learning-on-job* (phase III) consists of further autonomous education. This includes activities such as

[4] Under teacher training we understand supporting teachers' development of instructional practices.

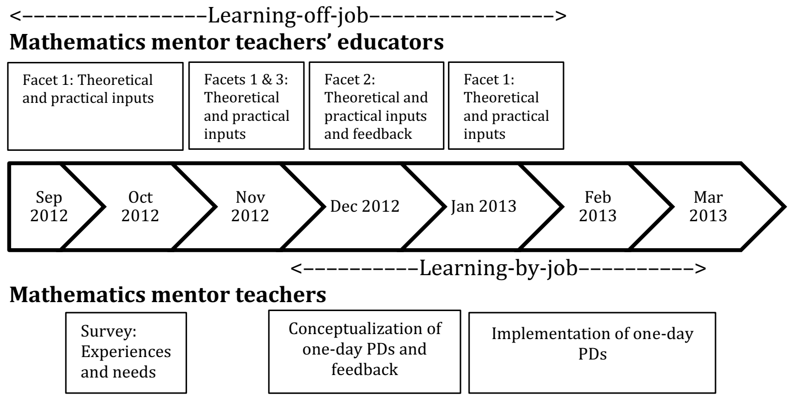

Fig. 1 Longitudinal model of pedagogical qualification of mathematics mentor teachers for designing teacher training in data analysis with digital tools

clarification of own mentor role, self-expectations, and needs for further development.

In the following we describe the adaptation of this model for our CPD based on the theoretical considerations, as outlined in Fig. 1.

Phase I (*learning-off-job*):

1. Facet 1: *Professional knowledge for teaching statistics*. Given mentors' lack of knowledge on systematic teaching of statistics, and, moreover, in designing PD in teaching data analysis with digital tools, which was confirmed by the survey (see Fig. 1), we invested an extensive amount of time on this facet. We will not describe this in detail in this paper, but see Biehler et al. (submitted).
2. Facet 2: *Professional knowledge for designing effective professional development courses*. Here we built on mentors' professional knowledge for designing PD. We gave a theoretical input and discussed the topics that are relevant when conceptualizing effective PD, such as adult learners and learning, factors that influence the success and impact of any PD, change process, and empowerment (Loucks-Horsley et al. 2003). When talking of factors defining high-quality professional development, we paid particular attention to six DZLM core dimensions and features of effective professional development (see Sect. 3). In addition, we made a distinction between CPD quality and structural features as suggested by Garet et al. (2001).
3. Facet 3: *Knowledge about models of teachers' professional knowledge for teaching statistics with digital tools.* We devoted one session to making the teacher model (Wassong and Biehler 2010) explicit, so that it could better serve the mentors as an orientation in designing their one-day PD. We wanted them to be aware of the complexity of teachers' knowledge, and to consciously select what they would consider the most important components for an introductory one-day PD. In addition we pointed them towards supplementary material and literature to extend the resources on which they could build their own future PD on teaching statistics.

Phase II (*learning-by-job*): Supervised conceptualization, implementation of a PD, and its reflection started after phase I. This phase included a development of a 4-h long PD on data analysis for mathematics teachers in teams of two to four. We gave them a lot of freedom in designing their own PD course, because we respected their professional role, interests, competences, and preferences. They were encouraged to choose the concrete statistical content for the PD according to their own preferences (facet 1), but having in mind the teacher knowledge model (facet 3). With regard to the adult learning methods, they could build on their own teaching experience, but needed to take into consideration DZLM design principles (facet 2). During this entire process they were supported and coached by the CPD instructors, collaborated with co-mentors, and continuously received feedback from CPD instructors. In this manner, the mentors were able to actively learn and consolidate their training skills.

Phase III (*learning-on-job*) did not take place within our course.

3 DZLM design principles and research questions

In the following, we provide our own elaboration and interpretation of DZLM's (2013) professional development design principles for the purpose of analyzing mentors' practices. The design principles are a guideline for designing and implementing all professional development courses within DZLM with different specificity depending on the topic, target group, and format of the professional development. For each design principle, we specify the research questions of interest taking into consideration the context of mentors' one-day PD in data analysis.

3.1 Learner-orientation

Under learner-orientation we understand an integration of participants' knowledge and experiences (Garet et al. 2001), and a focus on the heterogeneous and individual prerequisites and needs of participants (Clarke 1994). This has then direct repercussion on the content and methodological decisions (e.g., possibilities for collaborative work, exchange or presentation of ideas) (Lieberman and Pointer-Mace 2008). Thus, the participants are active members in a teaching–learning context of the professional development. This can take a number of forms, such as working on practice examples, engaging in producing classroom materials, or discussing how new materials or teaching methods can be used in the classroom (Garet et al. 2001).

With respect to learner-orientation, we were specifically interested in:

Given teachers' poor knowledge, issues, and concerns when teaching statistics, to what extent did the mentors

- align participants' needs and concerns with the PD, and how?
- align the content of the PD with teachers' individual practices, and how?

How were the teachers included into the active learning process?

3.2 Case-based learning

The change in teachers' teaching routines and practices can happen when new innovative ideas are presented in a manner that allows translation and integration into their actual teaching practices. For that reason, the content of professional development should be embedded into teachers' everyday teaching situations ("cases") by using practical experiences and examples from teaching practice (e.g., classroom artifacts). Moreover, participants themselves may bring their own case-examples into the professional development. Then so-called "cases" serve as both a starting point and a field of application for teaching and learning in the context of the professional development (Fullan 1995). These practical aspects should also be emphasized by using specific student artifacts, video clips or transcripts from teaching situations (Borko et al. 2011), which can then serve as a starting point for a context of application (e.g., diagnosis of students' misconceptions) or for a discussion (Lipowsky and Rzejak 2012). Thus, they serve as essential points of reference for the design of professional development courses.

With respect to case-based learning, we were specifically interested in:

- How did the mentors take the participants' practical experiences from the classroom, and to what extent?
- What kind of student work did the mentors incorporate, and to what extent?

3.3 Competence development

Competence development refers to improving and deepening participants' competences (e.g., content knowledge for teaching statistics) in order to create rich and high quality learning environments. This construct, as informed by empirical research (e.g., Wassong and Biehler 2010), is multifaceted, and includes mathematical content knowledge, pedagogical content knowledge, technological knowledge, and technological pedagogical content knowledge, among others. This competence and goal orientation should be transparent for all concerned parties, and should be clearly identified by the professional development leaders (Lipowsky and Rzejak 2012). On the one hand, it is important for the leaders during the professional development to make the learning coherent and goal-oriented (Elliot et al. 2009). On the other hand, this allows the participants to link the new ideas to their own teaching practice and to make the learning process tangible (Garet et al. 2001; Lipowsky 2011).

Given that we specifically focus on what facets of knowledge were covered in the PDs in the paper of Biehler et al. (submitted), here we were instead interested in:

- How and to what extent did the mentors make explicit the facets of knowledge that would be broached during the different phases of the PD?

3.4 Application of various instructional formats

To provide instances of the above-mentioned design principles, a variety of instructional formats is required. An intertwinement of (theoretical) input, active learning, and reflection phases is crucial for establishing connection between theory and practice (Lipowsky 2011; Lipowsky and Rzejak 2012). In addition, this principle is central for organizing practical phases (e.g., working in pairs or small groups, observation, sharing) during the duration of any professional development course.

With respect to various instruction formats, we were specifically interested in:

- Taking into consideration the format of the mentors' PD, what kind of instructional formats were supported and with what goal?

3.5 Stimulating collaboration

Stimulating collaboration refers to fostering participants' collaboration in order to foster the exchange of experiences. In the first step this can be achieved through activities such as sharing ideas, working together towards a common goal (e.g., implementation of a particular activity in own classroom), and reflecting on the learning process (*short-term collaboration*). The second and ultimate step is to continue the collaboration beyond the professional development course. For instance, this can be done through building professional learning communities (PLCs) of practice in which they jointly plan lessons and arrange mutual classroom visits (*long-term collaboration*). Consequently, such an environment supports reflection on own competences and practices (Putnam and Borko 2000). Here, it is important to note that this principle is dependent on the format of the professional development; whereas collaboration beyond professional development is a plausible requirement for long-term PD, short-term PD can achieve this only in a limited way (Cochran-Smith and Lytle 1999).

With respect to stimulating collaboration, and taking into consideration that some mentors' PD took place within their participants' schools and some took place outside, we were specifically interested in:

- What kinds of practices were used to establish and maintain collaboration during and after the PD, respectively?
- Is there a difference between collaboration practices in in-house and out-of-house PDs?

3.6 Fostering (self-)reflection

Relevance of professional development for the participating teachers and the sustainability of the professional learning can also be attained through reflective activities (Ingvarson et al. 2005). Participants are encouraged to engage in self- and collaborative reflection on covered topics/material and possible transfer into their own classroom as well as on their own teaching or training practice, student work and thinking, beliefs, and attitudes, to name a few (Putnam and Borko 2000). In particular, reflecting is regarded as a core feature of knowledge acquisition and skills development, and of effective change in practices (Cochran-Smith and Lytle 1999; Ingvarson et al. 2005; Lipowsky and Rzejak 2012). We contend, for that reason, that reflective behaviors should be prompted throughout the duration of the PD, and on different aspects, in order to expand their practices and consolidate their knowledge.

With respect to fostering (self-)reflection, we were specifically interested in:

- On what different aspects did the teachers get invited to reflect, and to what extent?

4 Methodology

4.1 Sample

Participants of the CPD program were 12 mentors, all active members of so-called "competence teams" in the federal state Nord Rhine-Westphalia (NRW). As members of a competence team they advise and support schools with regard to teaching mathematics, and offer on-demand PDs. This is done in about 20 % of their working time every week (1 day); they spend the other 80 % of their working time as regular classroom teachers (4 days). They were all experienced teachers with more than 20 years of teaching experience. Their training experiences, however, varied: some had over 10 years' experience as a mathematics mentor teacher, some close to 5 years, whereas some had just entered a competence team.

4.2 Data collection and instruments

All relevant data for this study was collected during the conceptualization, implementation, and reflection of the one-day PD undertaken by the mentors. We first outline the structure of the PDs, before describing the instruments.

Each team (2–4 mentors) developed a 4-h PD on a specific aspect of teaching data analysis with digital tools (see Online Resource 1). The general structure of the PDs was prescribed by the course designers and was composed of 4 blocks that also reflected effective CPD practices: (1) introductory block (ca. 1 h), (2) block 1 (1¼ h), (3) block 2 (1¼ h), and (4) reflection and closure (ca. ½ h). While the general function of the first and last block was clear, the mentors were free to organize and implement blocks 1 and 2, and could select content and activities they felt were helpful and relevant. We encouraged them to allow for opportunities for effective professional development practices (here DZLM design principles). Thematically, they were supposed to select from the topics covered in our CPD, and to use the software Fathom (for more detail see Biehler et al. submitted). They were, however, allowed to decide on the nature (demonstration vs. teaching/learning tool) and the intensity of the Fathom use. Each PD was attended by 8–14 mathematics teachers.

The mentors' development, implementation, and reflection of the five one-day PDs was documented using the following instruments: (1) documentation of the plan for the PD, (2) video data from the PD, and (3) reflection sheet on the implemented PD. These materials served as data sources for this study. In the documentation of the plan for one-day PDs, the mentors had to, among others, describe promoted facets of professional knowledge (Wassong and

Biehler 2010), what DZLM design principles were incorporated, and in what manner they would be addressed. The documentation was submitted before the PD took place. Within 2 weeks of the PD taking place, each team submitted a reflection sheet in which they reflected on the implemented PD. All five mentors' PDs were video-taped. This data was used to answer explicitly the first research question. On this basis, we were able to discuss and document practices that did not reflect our expectations or were missing, helping us answer our second research question.

4.3 Data analysis

For this study, a qualitative research design was used. Data analysis was performed in several steps in which we analyzed the documentation, the videos (total of 20 h), and reflection sheets from each PD. First we selected for examination parts from the video data that were of interest, and divided those into units of analysis (blocks of PD as described in Sect. 4.2). These were then compared with a coder from the team, and discussed until an agreement was reached. The selections were then analyzed using content method analysis as suggested by Miles and Huberman (1994), starting with PD 1. First we noted deductively instances in which the mentors engaged in PD practices related to DZLM's design principles. Through this process we classified the data first into the six categories. In the next step we looked for instances (i.e., subcategories) of how each principle was operationalized, as described in Sect. 3. This allowed us to build a codebook for PD 1, in which three columns were of interest: category, subcategory, and examples. After PD 1 was analyzed, we coded each PD using a similar method. The final step in the analysis was to merge the five codebooks, adding another column in which we assigned the examples to a particular PD. To supplement this data with regard to the research focus, the documentation and reflection sheets were analyzed using the above-described procedures. Based on our codebook (see Online Resource 2), another expert in the area conducted the data analysis independently. To ensure an acceptable degree of reliability, we checked the inter-rater reliability by using the formula recommended by Miles and Hubermann (1994) in which the coder reliability is calculated in the following manner: coder reliability = number of agreements/(total number of agreements + disagreements). The inter-rater reliability was calculated at 96.6 %.

5 Findings: characterization of the teacher training practices in five one-day PDs

In what follows, we present our findings by focusing on interesting aspects of each design principle against the framework outlined in Sect. 3, and how these were operationalized. Due to the complexity and diversity of our data, we can only present a subset of results in this paper.

5.1 Learner-orientation

As already said in Sect. 3.1, with respect to learner-orientation, we were specifically interested in:

> Given teachers' poor knowledge, issues, and concerns when teaching statistics, to what extent did the mentors
>
> - align participants' needs and concerns with the PD, and how?
> - align the content of the PD with teachers' individual practices, and how?
>
> How were the teachers included into the active learning process?

Aligning participants' needs and concerns with the PD. The participants' needs and concerns were aligned either implicitly or explicitly. The implicit alignment occurred by announcing topics and their rationale that would be covered in the PD abstract sent out before the PD. The topics were all part of the NRW curriculum, both from a content perspective (e.g., data representation, data analysis) and a process perspective (e.g., task culture, use of digital tools). Given the insufficient statistics education and new emerging tools for teaching statistics (e.g., Burrill and Biehler 2011), these topics were relevant to the mathematics teachers.

The explicit alignment occurred in only one PD by collecting data from the participating teachers in advance and making spontaneous decisions during the PD. Here the mentors sent a survey to the participating teachers and gathered the information about teachers' expectations for the PD, topics of interest, current statistics practices, and vision for teaching statistics. In this way the learner-orientation was increased, and reflected through topics, activities, and discussions (e.g., learning Fathom basics, interpreting boxplots, working on authentic tasks with reference to the curriculum) that were articulated by the teachers. In addition, the mentors also made spontaneous changes during the PD itself at the request of the mathematics teachers. In their written reflection, the mentors from PD 3 elaborated on this:

> Any changes made by us were based on the needs of the mathematics teachers or had an optimization of the learning process as target. We see the flexibility to react to the current situation as part of a professional implementation.

Thus, for these mentors, hearing the voice of teachers to support their needs was important.

Aligning the content of PD content with teachers' individual practices. All the PDs had a common focus on introducing Fathom as a new tool to teach statistics. From the teachers' perspective, none of them had experience of using Fathom when teaching statistics. At most they had limited experience with Excel. Therefore, we are interested in how Fathom was introduced in different PDs, and in particular how this was connected to the teachers' practices. All approaches were rather similar (see Online Resource 1), except in the case of PD 1. In all other PDs Fathom was introduced by using eFathom[5] without any reference to the teachers' previous practices in using Excel. In PD 1, however, in addition they got to carry out one activity in two ways (by hand and with Excel), and with Fathom. Afterwards they reflected on the three tools by evaluating the following characteristics: illustrativeness, expenditure of time, operability of the three, and tool preference. By this, the participating teachers were able to elaborate their pedagogical technological content knowledge.

In addition, it was discussed in PD 1 if and how Fathom could be integrated into their teaching practices. Thus, the innovation (here new software) was not introduced as separate from the teachers' previous practices, but as something they could build in into their teaching practices.

Active learning. The teachers were engaged actively in the learning process in all PDs. A balanced input-active-learning cycle took place, which created a constructive and open working atmosphere as a whole. A nice example of active learning took place in PDs 2 and 3 where the topic of boxplots was focused on; this topic is a source of problems not only for students, but also for teachers. Here the mentors took an approach rooted in embodied cognition, namely the "human boxplot"; that is, the teachers were carriers of statistical properties and had to sort themselves according to travel time to the PD and find quartiles and medians of the distribution. Through this activity the participants learnt how to construct a boxplot in a way that could also support students' learning. In addition, through another method of active learning—discussion—the teachers were given the chance to discuss among themselves and with the mentors the meaning of boxplots and how this approach may help students grasp boxplots and their interpretation. In other words, the mentors focused on specific subject matter, and how students may learn it to foster understanding—they carried the role both of a student and of a teacher.

[5] eFathom is a multimedia learning environment that offers a dynamic and example-oriented introduction to Fathom, included in Biehler et al. (2011).

As outlined here, we can see that the mentors' practices reflected those aligned with participant-orientation. This confirms the results of Rösken-Winter et al. (2015), where this principle was highest ranked in its importance. However, not all practices were good practices. Thinking back on our CPD for mentors, and on the basis of these results, there is a need to discuss more precisely how to foster participant orientation, and to give methods. While the mentors involved their participants in actively building new knowledge, this was rarely connected to their current practices and experiences, but rather was "injected into" them. This, according to Guskey (2002) and Loucks-Horsley (2003), may not have been enough for teachers to better their practices. In addition, the explicit alignment that seldom occurred requires the attitude to do this from the side of the mentors and the flexibility in knowledge and PD teaching method. Thus, we need not only to provide mentors with concepts of how to design needs-based PD, but also to challenge their views and attitudes about their role.

5.2 Case-based learning

As already said in Sect. 3.2, with respect to case-based learning, we were specifically interested in:

- How did the mentors take the participants' practical experiences from the classroom, and to what extent?
- What kind of student work did the mentors incorporate, and to what extent?

Using participants' practical experiences. This practice was taken into account in three different ways: by collecting information before, at the beginning, and spontaneously during the PD. Given that the first aspect was discussed in Sect. 5.1, we focus here on the latter two. In one PD, the teachers' experiences (e.g., use of tools, student learning difficulties and misconceptions, tasks) in teaching data analysis were surveyed at the beginning of the PD, and were an important part of the PD during the practical and reflection/discussion phases. In all PDs the mentors showed flexibility and responsiveness to mathematics teachers' questions about Fathom or implementation of the tasks in their mathematics classroom, as well as ideas with regard to developing and altering tasks and to differentiation possibilities. However, the questions directly addressing consequences for their teaching practices were answered either theoretically on the basis of our CPD or from their own practical teaching experience.

Incorporating student work. The mentors barely incorporated their pupils' work in the PDs. This is partly due to their limited teaching experience; however, examples they had encountered in our CPD course were also not used. Another explanation is the limited time, so that they decided to focus on content knowledge and technological content knowledge instead of pedagogical content knowledge. Student work was used only in two PDs, with different intensity and for different purposes. In PD 1, for instance, one of the mentors held in her hand printouts of students' work with Fathom. The mentor did this with the intention of trying to show the teachers that students can easily work on the problems using Fathom. In PD 2, photographs of student work were shown that demonstrated how pupils collected and analyzed data. This introduction was then used to suggest Fathom's use in school for analyzing and representing larger data sets. Hence, demonstration of pupil work served more as a bridge to motivate the affordances of Fathom when teaching and learning data analysis and not as material for detailed reflection on students' work.

The observation that examples from classrooms other than their own were not used may point to an important facet of mentors' attitudes—that their authority as a mentor is strongly based on their teaching experience.

The mentors had very limited teaching experience with Fathom, and for that reason had limited opportunities of collecting student work and using those in their PD. This had an immediate repercussion on our CPD for them—our structure did not allow for implementation of new ideas into their teaching. Hence, they should be given a chance to try out the their new competences in their classroom and collect examples of student work, which can then be discussed in the course, before using those competences in their own PD. This would have then enabled them to be able to support teacher learning and competence development in more depth.

5.3 Competence development

As already said in Sect. 3.3, with respect to competence development, we were specifically interested in:

- How and to what extent did the mentors make explicit the facets of knowledge that would be broached during the different phases of the PD?

While different mathematical knowledge facets were addressed with each activity in the PDs (Biehler et al. submitted), they were either explicitly or implicitly addressed. With respect to the former, the goals of an activity for teachers' learning were clearly verbally communicated in two PDs. Thus, the participant teachers were aware of the promoted knowledge facets, and hence the purpose of the activity. In other PDs this was done implicitly. In these, the teachers worked on examples without the promoted knowledge facets having been clearly communicated. This raises the question whether in those situations the activities appeared meaningful to teachers, or as "activity for activity's sake." Nevertheless, they were supported through a plethora of materials. These contained different tasks the teachers could use in their own classroom, but they were not didactically prepared. Only in one PD were technical handouts prepared, so that the teachers could be able to solve the tasks independently. Suggestions for how to implement worked-out activities happened when mentors themselves had used the activity in their mathematics lessons.

While we continuously made explicit in our course the content being learnt, and what dimensions of their knowledge would be enriched, this was not seen throughout all PDs. Neglecting making transparent the promoted competences and learning goals may end in so-called "implementation-dip" (Loucks-Horsley 2003). Most of the activities stayed on a "developing level" (Timperley 2011). In other words, practical ideas about new programs or innovations were presented and discussed. However, integration of in-depth knowledge about curriculum with how to teach was lacking, as was the attention given to student learning. We hypothesize that in-depth teaching experience in the domain of innovation is a necessary but not sufficient prerequisite to thoughtfully and thoroughly conveying the didactical ideas behind each innovation, as was also confirmed by Rösken-Winter et al. (2015). Thus, the PD is quite likely not to enhance teachers' understanding of the discipline or better their professional knowledge and practices.

5.4 Application of various instructional formats

As already said in Sect. 3.4, with respect to application of various instructional formats, we were specifically interested in:

- Taking into consideration the format of the mentors' PD, what kind of instructional formats were supported and with what goal?

All PDs implemented the structure we had provided: introductory phase, input–active-learning phase, reflection phase cycle. When exchange of experiences and expectations took place, this served as a means to clearly communicate the focus of the PD, and was again addressed after each active learning phase and during reflection phases. Such exchange also gave the participants the opportunity to share their issues with other colleagues and create a good basis for cooperation. Input talks served as a basis for teachers to be introduced to new curricular, teaching, and technological developments, which then were worked out during the practical phases. Here the mentors showed their knowledge of a plethora of methods (e.g., working in pairs or small groups, think-pair-share, stationary learning, observation, sharing). As already described in Sect. 5.1, stationary learning was used to allow teachers to compare the three statistics software tools and in that manner to build on their existing teaching practices. An additional innovative method, specific to statistics, was creating a human boxplot (see Sect. 5.1), which was used to both help teachers better understand the boxplots and their analysis as well as to learn a method to support students' learning. See Online Resource 1 for additional teaching and learning opportunities implemented in the PDs.

5.5 Stimulating collaboration

As already said in Sect. 3.5, with respect to stimulating collaboration, we were specifically interested in:

- What kinds of practices were used to establish and maintain collaboration during and after the PD, respectively?
- Is there a difference between collaboration practices in in-house and out-of-house PDs?

The collaboration was stimulated during the PD. This was done through joint activities, sharing, discussions, joint reflection, and by participants helping each other when getting stuck. However, the activities themselves did not target collaboration. For instance, the activities did not prompt teachers to discuss jointly implementation of a particular activity in their own classrooms. Further, collaboration beyond the PD itself was not stimulated. Whereas in the two in-house PDs this would have been possible, such as through creating professional communities and support systems by the mentor, this did not occur. We hypothesize that such collaboration did not happen for three reasons: building PLCs was not covered in our course, participants' limited experience of teaching with Fathom, and the format of the PD did not necessarily allow for starting a long-term collaboration.

While short-term collaboration was promoted, collaboration in the long term was not. This was partly influenced by the PD format, but may have also been a result of participants' understanding of their role as a mentor. Given that long-term collaboration is an important factor in sustainable PD (e.g., Zehetmeier and Krainer 2011), the question arises how such collaboration can be supported in short CPDs (both in-house and out-of-house PD). In our CPD we ourselves did not focus on concepts fostering long-term collaboration (e.g., PLCs). Therefore, our revised version of the course should address these two issues.

5.6 Fostering (self-)reflection

As already said in Sect. 3.6, with respect to fostering (self-) reflection, we were specifically interested in:

- On what different aspects did the teachers get invited to reflect, and to what extent?

Current teaching practices. During the PD the participants reflected on their current teaching practices, which were to serve as a basis for discussing the new ideas. These reflective practices were administered during four possible points in time: before the PD took place (in written form with semi-structured questions), at the beginning of the PD, during, and at the end of the PD (open questions). These reflections served both as starting points for structuring the PD itself and as discussion topics. The reflection at the end of the PD was part of every PD, allowing the teachers to discuss possible new practices with respect to previous experiences. The three other reflection points were part of only two PDs.

Tasks. The participants were also prompted to reflect on the given tasks. When such reflection was prompted, then it was administered through handouts that contained the task that had to be carried out followed by reflective questions. The reflective questions focused on three different components: competences, used representation and possibilities for other representations, and differentiation possibilities.

Transfer of ideas into the mathematics classroom of the participants. In all PDs the teachers were explicitly asked to evaluate the transferability of new ideas, tasks, and software into their own teaching. Depending on the PD, this occurred immediately after the idea or activity was worked out (in two PDs) or just once at the end of the PD. Thus, different mentor teams assigned this principle different relevance.

As noted, the mentors supported teacher reflection on different objects; however, they did not always do it throughout the PD. Moreover, reflection on student work and thinking, and beliefs and attitudes, did not take place. This raises the question of sustainability of teacher competence development, and change in teaching practices (Ingvarson et al. 2005; Lipowsky 2011). The results are aligned with those from Rösken-Winter et al. (2015) where the mentors assigned only moderate relevance to this principle for their own work and for their own PD design. This raises the further question of whether the mentors recognize the value of reflecting on different constructs, and possess the skills and techniques to do so. Thus, when revising our course we need to focus on: (1) facilitating reflective behaviors, (2) exemplifying theories and methods for reflection in one-day PDs, and (3) allowing for opportunities to show the additional value of reflection within PDs.

6 Discussion and further work

In this under-studied area, it was a challenge to create opportunities for developing a solid CPD design concept that would enable supporting of mentors in their complex role. More particularly, focusing and balancing learning in the two domains of theoretical knowledge (professional knowledge for teaching statistics and for designing effective professional development courses), and creating opportunities to elaborate, integrate, and apply this knowledge as a practical competence for designing and implementing PD in teaching statistics with digital tools, was challenging. Our results show that, though the mentors exhibited many of the practices aligned with effective high-quality CPD, they did not fully reflect on them. It seemed that, when designing and implementing their PD, the mentors focused more on the structural elements than on the quality PD practices across activities and different blocks of the PD. We therefore doubt the extent to which the activities supported the participating teachers' learning in the way that the mentors had intended.

We contend that the following factors accounted for the variability of implementations and limited implementation of design principles: our CPD program; the mentors' attitudes and knowledge, and how they made sense of our course for themselves; the boundary condition of the one-day PD they had to design; and the participating teachers' attitudes, competences, and interests. We elaborate on these in the form of questions and comments, and offer possible modifications for our CPD.

How important is own teaching experience in a content area that mentors focus on in a PD compared with other sources of authority in PD teaching?

As we saw in Sect. 5.2, our mentors tend to use only examples from their own teaching, and not from the literature and other teachers' classrooms. It would be helpful to give opportunities for gaining own teaching experiences during the CPD course itself. However, this has practical limitations as most of the participating mentors did not have a class where statistics was taught. It would have been much easier if the topic of a PD course had focused on teaching and learning methods. Therefore, we may also have to change mentors' attitudes with regard to using other sources of authority than own experiences. We may also have to more systematically include vivid examples (e.g., videos, student materials) from other teachers' classrooms that can then be used by the mentors more easily.

How can we improve the phase "*learning-by-job*"*?*

If we take mentors' PD practice as a point of reference, we may wish to apply the principle of case-based learning also to the mentors' learning. Ideally, documents and episodes of real PD courses could serve as a "case" to illustrate DZLM design principles as well as features of successful PD courses on learning statistics with digital tools. We required our mentors to create their own PD course from the knowledge sources we provided (facets 1, 2, and 3). We did not provide scripts and structure for such a one-day course ourselves (although that would have been possible), because we did not want to "prescribe" too much. After the first round of our CPD course, we now have five examples of PDs from which we can draw examples and episodes, whose analysis can help future mentors to better plan their own PD. Maybe the idea of collaborative lesson studies, which has been proved to support teachers' competence development effectively, should be applied to a course for mentors as well, meaning that collaborative PD study could become an issue not only in the planning stage (as we did) but also in the implementing stage. This, however, has many practical limitations, as it is difficult to have participating observing mentors in a PD in addition to those mentors who are leading the PD. Moreover, fostering self-reflection within a CPD course for mentors will have to be enhanced, too. The PDs implemented by the mentors took place after the end of our module. It would have been better to have one or two more meetings after the mentors' practical experiences in order to stimulate collective reflection.

From analyzing the mentors' practice in the five PDs, we found quite a few instances that required much more competence and flexibility from the mentors with regard to statistics education and with regard to PD teaching than could be observed in action. We based our course on a preliminary "normative job analysis" of mentors' objective needs. However, we were not always successful in communicating this to our mentors. An analogy is how to motivate teachers to study the subject matter of mathematics as such in order to become more competent teachers. Cases from the classroom that require flexibility in knowledge and reaction are a means for motivation. In the future, we may also wish to use cases from PDs in order to better motivate and refine the knowledge we offer in the course to our mentors. This was not possible in the first implementation, but when we redesign this mentor course, we have several concrete cases we can build on.

As suggested by Wahl et al. (1991), we further contend that a "sandwich model" in which different learning phases cycle should be used when designing CPD for mentors. However, we suggest doing it integratively, that is, after each subtopic, if possible. Thus, a role change after each subject-matter topic (student–teacher–mentor) would allow participants to slowly grow in their competence development, gain teaching experience, and gather advice from other colleagues. Thus, they would build integratively on their competences through alternation of theory, teaching, and training practices.

Last but not least, we found that some of our design principles for successful CPDs are not part of the PD culture in which our mentors have to work in the future. The designers of CPD for mentors need to address this discrepancy; if it is not possible to change this culture, at least the participating mentors have to be made aware of the obstacles they will encounter.

We believe that these changes will result in more successful support for mentors' development of effective PD courses and practices, which would allow for high-quality and effective large-scale dissemination.

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