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Developing mathematics teachers' competence through parallel lesson study

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Abstract

Purpose – The purpose of this paper is to examine practicing mathematics teachers' learning through parallel lesson study in China. Lesson study in China has been practiced for decades. Parallel lesson is an enriched mode of lesson study to address the implementation of new curriculum.

Design/methodology/approach – The expansive learning perspective has been used to explore the ways practicing teachers learned to improve teaching through the transformation of learning objects and boundary crossing.

Findings – Two cases are illustrated and compared to highlight features of teachers' learning through parallel lesson study. The practicing teachers developed their competence in transforming instructional objectives and task selection and implementation. In addition, they also developed professional vision in alignment with the reform-oriented curriculum.

Originality/value – This study makes significant contribution to understanding teachers' learning through lesson study in China. Meanwhile, it also demonstrates how the theory of expansive learning could be used as a conceptual framework to examine teachers' learning through lesson study.

Keywords Expansive learning, Lesson study, Parallel lesson study, Teacher professional competence

Paper type Research paper

Introduction

In China there is a well-established, multi-tiered teaching research system through which teachers and teaching researchers (subject specialists) work together to design, deliver, and revise lessons to promote high quality student learning (Huang *et al.*, 2010; Ma, 1999). Chinese lesson study is a common and core component of various professional development mechanisms (Yang and Ricks, 2013), which provides opportunities for teachers to develop professional knowledge that links theory to practice (Kieran *et al.*, 2013). Although some researchers have explored how Chinese teachers improved their teaching practice and developed their expertise through conducting public lessons (Han and Paine, 2010) and exemplary lessons (Huang *et al.*, 2011), the literature on deep understanding of teachers' learning process is still scarce. With the implementation of new curricula (Ministry of Education, P. R. China, 2011), Chinese lesson study has evolved into parallel lesson study, which enriches the traditional lesson study by

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contrasting one exemplary lesson with another one, respectively, developed by two different lesson study groups (LSG) at the same time. Thus, an in-depth analysis of the enriched Chinese lesson study process will provide insights into deep understanding of mathematics teachers' learning and designing effective professional development. Specifically, this paper addresses the following research question:

RQ1. In what ways do practicing teachers develop their professional competence through parallel lesson study?

Background and theoretical framework

An enriched Chinese lesson study: parallel lessons

The practical philosophy that teachers' professional knowledge and competences can be improved through studying lessons is fundamental in China, and underlies all teaching research activities. The Chinese Lesson Study refers to a model of professional development that includes cycles of collaborative development of lesson plans, delivering lessons along with classroom observation, post-lesson debriefing and reflection, and revision (Huang and Bao, 2006; Yang and Ricks, 2013). Similar to the Japanese Lesson Study (e.g. Lewis, 2002), in terms of their activity structures, the Chinese lesson study model pays much attention to "both content and pedagogical knowledge and skills, and an open, learner-centered implementation component" (Lerman and Zehetmeier, 2008, p. 139).

Recently, parallel lesson study (PLS), an enriched Chinese Lesson Study, has become a very popular model. The PLS approach is a response to the call of new curricula that requires teachers to creatively and innovatively use their textbooks in their classrooms to provide differentiated instruction for their students (Li, 2009). To conduct a PLS, a key topic is selected based on extensive discussions among teachers and mathematics teaching researchers. A LSG usually consists of a mathematics-teaching researcher from a district educational bureau, a master teacher, a demonstrating teacher (who takes the main responsibility for developing and teaching the selected content), and other mathematics teachers. Through a typical process of PLS, at least two independent LSGs develop exemplary lessons of teaching the selected content. Then a teaching research activity at the cross-district level is organized, inviting teachers from different study groups to demonstrate their respective lessons. A post-lesson meeting focusses on comparing and contrasting the public lessons.

Expansive learning and teacher professional development

Various theories have been utilized to examine teachers' learning, such as constructivist perspective and social cultural perspectives. Recently, sociocultural theories including community of practice, activity theory, and post-modern theory have been drawn on to investigate teacher change (Goos and Geiger, 2010). In this paper we adopt the theory of expansive learning to conceptualize PLS as an approach to mathematics teachers' professional development. The theory of expansive learning focusses on the transformation of learning from isolated individuals to collectives and communities (Engeström and Sannino, 2010). It argues that learning is a multidimensional process in which learners construct new objects for their collective activities and implement the new objects in practice.

The study employs the key concepts of expansive learning: transformation of the object, boundary crossing, and community building. Expansive learning is manifested primarily as changes in the object of the collective activity. The changes in the object could be featured by qualitative turning points (Engeström and Sannino, 2010).

The concept of boundary crossing has been used to characterize expansive learning that takes place in networks of multiple activity systems. Boundary crossing refers to "enter onto territory in which we are unfamiliar and, to some significant extent therefore unqualified" (Suchman, 1993, p. 25) and "face the challenge of negotiating and combining ingredients from different contexts to achieve hybrid situations" (Engeström *et al.*, 1995, p. 319). The sections that follow provide detailed descriptions of how these lenses were used in the study.

The job-embedded approach to professional development enables mathematics teachers in China to learn and improve teaching within a professional community of practice (Yang and Ricks, 2013). In this study the teachers and teaching researchers collaboratively worked on developing research lessons to pursue excellence in teaching. Their school-based teaching research groups (TRGs) and district level TRG formed their communities of practice (Wenger, 1998), in which they were engaged in the joint enterprise of parallel lesson development and developing their shared repertoire of knowledge and skills for teaching mathematics. Through active participation in their communities of practice, they can envisage the world of other teachers' practices and thus through extrapolating from their own practice they enter further into participation within their communities of practice. Essentially, they attempt to achieve the goals of effective teaching that is commonly agreed upon by all the participants in the communities of practice.

Learning is often identified in changes in learners' beliefs, behaviors, and cognition from a cognitive perspective. In expansive learning, construction of a new object and changes in the object of the collective activity manifest participants' learning. "In successful expansive learning, this eventually leads to a qualitative transformation of all components of the activity system" (Engeström and Sannino, 2010, p. 24). Teaching is a complex practice, from establishing instructional goals and planning out ways to deliver instruction, to assessing student learning. How to teach a mathematics topic effectively cannot be prescribed by theories, principles, or common sense. It is an object that must be constructed and transformed by local teachers based on their local theories. In this study, the two PLS groups embarked on a process of designing and implementing new ways to teach the topic of finding patterns in calendars and using symbols to express the patterns. We aimed to understand how the PLS activities transformed the research lessons to become "new objects" for teacher learning.

Wenger (1998) explained that as communities develop, they create boundaries between participants and non-participants. Akkerman and Bakker (2011) proposed that boundaries between communities establish "the potential difficulty of action and interaction across the systems but also represent the potential value of establishing communication and collaboration" (p. 139). Thus, boundary crossing creates spaces that are part of different communities. The members of different communities have "something to do together, some productive enterprise around which to negotiate diverging meaning and perspectives" (Wenger, 1998, p. 114). In addition, Wenger (1998) also conceptualized the notion of boundary brokers – those individuals who can introduce elements from one community into another. The teaching researchers and teachers represented different communities. The teaching researchers coordinated the parallel lesson study activities and collaborated with the participant teachers to generate new teaching approaches for a common goal. The participant teachers brought the new learning back to their own school-based community of practice. In this study, we employed the concept of boundary crossing to understand the participant teachers' learning through parallel lesson study. The parallel lesson study activities were a process of boundary crossing as shown in Figure 1.

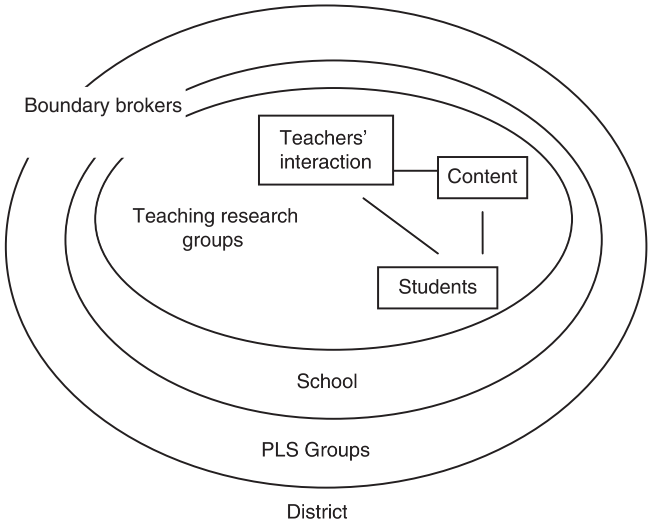

Figure 1. Boundary crossing among multiple communities of practice

As shown in Figure 1, the school-based TRG forms a community of practice, where teachers work together to develop research lesson based on the group's understanding of effective ways to teach the topic to their students. Outside the school, the parallel LSG extends the school-based community of practice to include teachers from different schools and knowledgeable experts from the district-master teachers and teaching researchers. Across these two communities of practice, teaching researchers play boundary broker roles when supporting teachers to conduct PLS activities. The lesson study design – repeating the cycle of planning, teaching, and observing, and debriefing and reflections offers rich opportunities for teacher learning and improvement of practice. Finally, the exemplary lesson from each PLS group functions as "boundary object" to make boundary crossing between two PLS groups possible. In the theoretical lens of expansive learning, the teachers in the current study constructed parallel research lessons (exemplary lessons) while focussing on creating a new approach to teaching a math topic that has been recently recommended in curriculum, and effectively implementing the new approach in practice.

Methods

Setting and participants

LSG A was supervised by Mr Zhu, a mathematics teaching researcher (MTR) in charge of teaching research activities at the middle school level in District A. Mr Zhu was an experienced teaching researcher, with a bachelor's degree in mathematics and a senior professional rank, 15 years of teaching experience in middle school and seven years of service as a MTR. Mr Wu was the demonstrating teacher in Group A. Mr Wu possessed a master's degree in mathematics and had just started his first year of teaching.

Mr Hu, a teaching researcher in charge of teaching activities at both middle and high schools in District B, formed LSG B. Mr Hu, considered a novice MTR with three years experience, was previously an excellent senior teacher with a bachelor's degree in mathematics. Miss Han, the demonstrating teacher in LSG B, had a bachelor's degree in mathematics, with an intermediate professional rank and six years of teaching experience.

These two LSGs focussed on teaching an activity-based lesson in seventh grade to explore patterns in calendars and use algebraic expressions to express patterns. It was the first time that the students learned algebraic expression formally. An activity-based lesson, a newly recommended instruction model in the new curriculum (Ministry of Education, P. R. China, 2011), aims to get students engaged in mathematics activities that include the process of observation, experimentation, conjecture making, justification, and communication based on their existing knowledge and experience.

Data collection

The data of the current study came from a larger study (Huang *et al.*, 2014) that included five data sources:

(1) all the lesson plans created during the process;

(2) the teachers' reflection reports on learning from participating in the PLS;

(3) videos of the final public lessons;

(4) interviews with the two demonstrating teachers; and

(5) interviews with the two teaching researchers.

This data was collected over three months during the fall of 2012.

Data analysis

A grounded theory approach (Corbin and Strauss, 2008) was used to analyze the multiple sources of data. Based on the lesson plans created for each rehearsal lesson and the final research lessons, the researchers identified critical changes of the lessons over time that were considered turning points, which formed the base for interview prompts to understand the teachers' recognition and interpretation of those turning points. With the interview data, triangulated with the demonstrating teachers' reflection reports and transcribed videos of the final research lessons, the researchers further classified two turning points: instructional objectives and mathematical tasks. In addition, the researchers discussed these turning points with the two teaching researchers, who confirmed these findings.

Meanwhile, what the teachers learned from conducting the PLS activities was also examined through the conceptual lens of boundary crossing. Based on the analysis of the turning points in the lessons, the researchers drew on the data of the interview and the reflection reports to understand how boundary crossing influenced the teachers' learning. Besides the turning points of the lesson transformation, the participant teachers' perspectives were heavily influenced when the teachers and the teaching researchers crossed the boundaries of communities. Two remarkable changes were identified due to discussions and negotiation of different ideas in multiple communities, including professional vision on learning and awareness development of improving teaching.

Results

The results are presented in two sections. First, we present teacher learning through the transformation of lessons. Then, we describe teacher learning through the conceptual lens of boundary crossing.

Transformation of the lessons and teacher learning

Throughout the process of conducting lesson study activities, the research lessons (See a brief description of the research lesson in Appendix) have been transformed to aim at promoting student learning from mastering solving specific problems to discovering the methods of finding patterns through solving problems. During the process, the teachers' learning occurred in a recursive way when they revised and polished their instruction with the support of colleagues and more knowledgeable experts (the teaching researchers).

Turning point one-transformation of instructional objectives. The teachers had no experience in teaching activity-based lessons, which stresses discovering, reasoning, and communication (Ministry of Education, P. R. China, 2011). It is critical to develop their understanding and implementation of the innovative instructional goals.

Both teachers in the PLS activities shifted their instructional goals from content-focussed toward process-oriented goals by improving teaching consistently. Miss Han expanded the original lesson objective of "focusing on finding patterns" to "focusing on the methods of discovering and expressing patterns, and mathematical methods underlying these processes of discoveries" (Interview). With feedback from Mr Hu, the teaching researcher in her district, Miss Han realized that she needed to shift her focus from content and skills to process and methods, and disposition and attitudes. The focus was on how to find invariant patterns (e.g. the sum of these consecutive numbers equals the middle number times the total number of the grid) through examining varying grids (a row, a column, a diagonal, or an X grid). As stated in her interview, "Finally, I specified overall instructional goals: the process objectives became more important while the knowledge objective become less important." Miss Han also realized the importance of aligning her actual teaching with the lesson objectives due to the teacher researcher's consistent reminders. In her interview, Miss Han said, "He often asks me what my instructional objectives are. Did I stick to these objectives?" Mr Hu suggested that it was not sufficient to let students master using letters to represent numbers and seeking patterns. The objectives should include the experience of making connections among different concepts, and the strategies of identifying patterns. Based on these suggestions, Miss Han further clarified the instructional objectives and fulfilled these goals throughout the entire process of PLS.

Similarly, Mr Wu developed comprehensive instructional objectives based on the colleagues' and teaching researcher (Mr Zhu)'s comments, and self-reflection on repeated rehearsal lessons. At the beginning, he focussed on finding the patterns. Through revising and teaching the rehearsal lessons, he gained a better understanding of how to set up instructional objectives. After the second rehearsal lesson, the teaching researcher gave Mr Wu very concrete suggestions on how to summarize patterns in Activity 1, and how many diagrams should be presented to students (at least three for each case). Specifically, he emphasized the importance of using letters to represent numbers. Mr Wu gradually re-geared his lesson toward developing students' strategic competence so that students could learn different solution methods to identify patterns and appreciate the power and usefulness of expressing quantitative relationships using letters. As explained in his reflection report as follows:

> Mathematics lessons should focus on the process of exploring and acquiring, rather than understanding the knowledge itself. The process not only can lead students to better understanding of knowledge, but also can lead students to think mathematically when doing activities, experience the value of knowledge, increase the awareness of application of mathematics knowledge, and experience the connections between mathematics and daily life.

Turning point two – transformation of mathematics tasks. Task selection sets the tone of the lesson and can focus student learning on high-level cognitive demand and engage students in thinking (Stein *et al.*, 2000). Student engagement is an important aspect teachers need to consider when planning lessons and assessing student learning (Ministry of Education, P. R. China, 2011). Strong engagement in learning will support students to develop disposition of persevering in problem solving. With her colleagues' and teaching researcher's suggestions, Miss Han changed one of the mathematics tasks in the lesson. In the interview, Miss Han mentioned a commentary offered by her colleagues and Mr Hu (the teaching researcher) that the introduction of the original second task did not prepare students to solve the major task. In the original second task, Miss Han showed a frame with three numbers and asked students about the relationship among the three numbers. Her colleagues observed that the students did not have a chance to think and raise their own questions. Instead, the specific example was abruptly imposed onto them. Taking the feedback, Miss Han designed a game in which the students chose three consecutive numbers in a row and told the teacher the sum of the three numbers, and then the teacher quickly figured out what the three numbers were. The game motivated the students to make conjectures and discover patterns that explained how Miss Han could list the three numbers so quickly.

To design and implement the major exploratory task, Miss Han attempted different methods to provide grids for students to explore: initially selecting two grids from six grids, then experimentally selecting simple ones from given grids, and finally providing two selected grids. However, she realized the final selection constrained student thinking. In addition, she changed the language from "finding the relationship among the three numbers" to "finding the arithmetic relationship[1] among the three numbers" to clarify the task.

Beyond setting the exploratory tasks for developing the new knowledge and methods, Miss Han also created a challenging task, arrangement of odd number, which required students to apply the learned thinking methods to a novel situation. In the interview (Han, 12-19-2012), she said:

> I completely agreed with my colleagues and Mr. Hu that the goal of the lesson is to help students understand the methods of seeking patterns. Beyond the calendar context, many similar application problems are not in natural number sequences, but in arithmetic sequences [in high school mathematics]. So, I designed the arrangement of an odd-number sequence to ask students to explore similar problems using similar methods. Thus, it can develop students' ability to apply what they learned. It emphasizes what they learned are not only applied to calendar but also to other situations.

Furthermore, Miss Han explained how Mr Hu helped her polish the task focussing on the learning goals of this lesson:

> He (Mr. Hu) believed that this is a good task but it may be too challenging for middle graders. In an initial design, the task included four sub-tasks. During the rehearsal lessons, I felt the same way. One of the sub-tasks was related to solving equation. Mr. Hu believed that the major goal of the lesson is to seek patterns and I could choose to not address equations. Eventually, I only used one of the four sub-tasks due to time constraint. Generally, the more I thought the more I wanted to cover. However, Mr. Hu always reminded me that I should focus on instructional goals. In particular, the goals should be the process of seeking patterns and methods of discovering patterns while the application of the patterns should not be the focus.

In his reflection report, Mr Wu described the process of changing his lessons in great details. Mr Wu drafted five versions of the lesson plan during parallel lesson study activities. Across the five drafts he kept modifying the mathematical tasks with colleagues' and Mr Zhu's feedback. After discussing the first draft of the lesson plan with his colleagues, Mr Wu adopted only one task from the textbook (mathematics patterns in calendar) because of the students' prior knowledge and lack of experience in problem solving. After their observation of the first rehearsal lesson in Mr Wu's class, his colleagues suggested adding some simple scaffolding tasks such as exploring patterns in a row, a column, and a diagonal with concrete numbers before exploring the major task. They also commented that in an activity-based lesson, the teacher should let students engage in activities as well as express their ideas. Mr Wu revised the lesson plan to include three phases of tasks that provided scaffolding to support students in proving their conjectures algebraically. In the third draft, Mr Wu designed three phases of activities. Phase 1 involved solving the simple problem of finding missing numbers in grids (two or three little squares in a row, column, or diagonal). At Phase 2, the students discussed and summarized the patterns embedded in a row, a column, or a diagonal and justified their conjectures algebraically. At Phase 3, the students explored the patterns of 3×3 grids within a calendar progressively. However, at the debriefing meeting, Mr Wu's colleagues pointed out that the lesson did not highlight the main goals – the reason and advantages of using letters to represent numbers.

With the feedback and his own reflection, Mr Wu kept revising the lesson plan. In the fourth draft, he added a table at Phase 1 to record verbal and mathematical languages to present the patterns. He aimed to help students experience the process of using letters to represent numbers from specific cases to general situations. At Phase 2 he had the students work in groups to make and justify conjectures by using algebraic knowledge. At Phase 3, after exploring two specific 3×3 grids, he asked the students to explore patterns in general and prove them algebraically. He taught the fourth draft of the lesson plan in a public lesson. Unexpectedly, the students immediately figured out and justified the patterns algebraically when the first 3×3 grid task was presented to them. In the final version of the lesson plan, at the Phase 3, he kept only one 3×3 grid task while creating a "cross" grid problem and a 4×4 grid problem for further exploration. Teaching through varying problems is a common strategy to promote students learning in China (Gu *et al.*, 2004). Mr Wu implemented the principle of practicing with variation problems for developing students' mathematical reasoning. When drafting and revising the lesson plans, Mr Wu and his colleagues adjusted and adapted the tasks based on the students' learning and reactions. They attended to both types of variations with math problems – conceptual and procedural variation.

Specifically, Mr Wu explained how he constructed appropriate scaffolding tasks and deconstructed inappropriate scaffolding problems (Interview with Mr Wu, 12-15-2012) with the support of the teaching researcher as follows:

> In our earlier designs of the lesson, students explored the patterns in the first two tasks adopting the inductive method. We designed the third task (Pattern in 3×3 grid) adopting the same method [i.e. discussing two specific 3×3 grids to find the pattern, then encouraging students to prove the patterns algebraically]. However, in one rehearsal teaching, students directly presented the patterns using an algebraic expression. The teaching researcher suggested the exploration of the two specific cases could be deleted. I also felt it is unnecessary to explore the concrete cases, particularly for high-achieving students. So I deleted these specific cases in the final lesson and encouraged proving the pattern deductively.

Boundary crossing and teacher learning

The PLS activities wove the school-based TRGs, the district-based PLS groups, and the cross districts PLS groups into a learning community for the participant teachers. Traditionally, mathematics teachers in China have demonstrated more consensus regarding best instructional practices in mathematics (Paine and Ma, 1993; Li and Huang, 2013). For example, they have been working under the framework of teaching and learning key points and difficult points (Yang and Ricks, 2012), emphasizing basic knowledge and skills, using accurate math language, etc. (Li and Huang, 2013). However, working in the more diverse learning community of the PLS might lead teachers to different dynamics and ideas in the learning communities.

The two district teaching researchers, Mr Hu and Mr Zhu, had more experience in teaching and research on instructional practices. Both of them won many awards for their excellent instructional practices in mathematics and had rich experiences in coaching young teachers and research on teaching. When they facilitated the PLS activities, they brought their knowledge, experiences, and views into the PLS demonstrating teachers' learning communities. Their boundary crossing back and forth from the districts' teaching researchers' communities to the school-based PLS groups resulted in negotiation of diverging ideas and perspectives, and transformed the research lessons and teacher learning. Moreover, the teachers developed their awareness of improving their teaching and pathways to their professional growth through comparing and contrasting the final exemplary lessons.

Developing professional vision. Miss Han's views on mathematics teaching and learning were reshaped during the process of conducting the PLS, which has a long-term impact on her professional development. When reflecting upon her own learning during the PLS, Miss Han said that she had developed a deeper understanding of the goals of mathematics teaching and learning with the support of her mentor, Mr Hu:

> The essential goal of mathematics instruction may not be content points […] but letting students understand how to realize the importance of using letters to represent numbers through solving daily life situation problems, and how to apply algebra to solve real-world problems. In other words, helping students understand that mathematics comes from life, for life, may be the essential goal.

To echo the goals of mathematics teaching and learning, Miss Han changed the lesson objectives for the research lesson which emphasized the process and methods rather than mathematics content knowledge. She appreciated Mr Hu's comments that her research lesson should focus student learning on building connections among different concepts and digging deeply into the thinking methods of discovering patterns rather than the mastering of using letters to represent numbers and seeking patterns.

Another professional vision Miss Han developed from conducting the PLS was about the relationship between instructional design and instructional implementation. Teaching is such a complex practice that is affected by multiple factors. When teachers plan out lessons as a body of knowledge and skills, and an attempt to achieve learning consequences, they would not be able to guarantee that the lessons would be implemented as planned. Faced with this discrepancy, teachers may need to modify lesson plans to respond to student reactions that occur during the implementation process. Miss Han gained an important vision on this critical relationship between lesson design and lesson implementation from the teaching researcher, Mr Hu. She understood that lesson design was intended design. When implementing the intended design in the classroom, teachers must take student learning into account and adjust to the classroom situation. Mr Hu told her as follows:

> Teaching designs are only "intended designs," and you can improve them endlessly. Others can always critique your design. You can purposefully adopt different ideas to develop a reasonable instructional design. However, when you implement your design, no matter how

> you formed your design (reflecting on your lessons, or adopting others' comments), you have to seriously consider students' learning, asking questions such as how can I implement for these students, how can I guide them to learn effectively in the class." (Interview with Miss Han, 12-19-2012).

Mr Hu helped the teacher develop a healthy, flexible view on intended curriculum and implemented curriculum, which would eventually help her to be open-minded to colleagues' critiques.

Mr Wu deepened his views about teaching, learning, and classroom instruction. His new research lesson objectives reflected this shift, which was discussed above. In his reflection journal, Mr Wu further illustrated his three major shifts in his visions about mathematics instruction (e.g. teacher roles, student roles, and a good lesson). He believed that teachers should have transferred their roles from "being knowledge transmitters to being organizers, guides, collaborators, and co-learners of students' learning by listening to students". Meanwhile, students should become "active learners and learn how to learn, master textbook knowledge, and discover knowledge through the acquisition of mathematics". "An excellent mathematics lesson should have the following features – fluency, openness, collaboration, and goals-oriented instruction, which are achieved through classroom discourses and peer collaborations during the process of solving problems".

This new perspective of teaching and learning had been developed through multiple rehearsal teachings with the support from experienced teachers and the teacher researcher. For example, after watching Mr Wu's rehearsal lesson, the teaching researcher, Mr Zhu, reflected on the issues in Mr Wu's lesson in his blog:

> The teacher acts likes a tour guide, leading students to visit Grand View Garden regarding knowledge. The students greatly appreciated the teacher's strong foundation. But, from a students' perspective, they maybe, like me, became more and more confused. At the very beginning, the teacher led students to explore the patterns of three consecutive numbers in a row […] and then in a diagonal, and then in a 3×3 grid diagram and so on. My confusions include: why do we need to study the patterns with three, or five or seven consecutive numbers, rather than the patterns with two, four and six consecutive numbers? […] They only know to explore the issues that the teacher designed by following the methods that the teacher demonstrated mechanically. It is only to do some imitation, not favorable for high-order thinking ability development.

Mr Zhu's observation depicted a traditional way to teach activity-based lessons, which was on the opposite of what Mr Wu eventually realized about excellent mathematics instruction. In his rehearsal lesson, Mr Wu was still the knowledge transmitter, while his students did not take part in the learning discourse to collaborate and construct knowledge actively. Mr Zhu's mentoring across boundaries helped Mr Wu reframe his visions on excellent, innovative mathematics instruction. With the support of experienced teachers and the teaching researcher, Mr Wu successfully improved the research lesson by establishing appropriate scaffoldings (using systematic and diverse grids), demonstrating the power of using letters to represent numbers, and building the connections between mathematics and real life situations.

Developing awareness of and pathways to improving teaching. In a cross-district teaching research activity, Miss Han and Mr Wu delivered their lessons in the same school to two different classes back to back. They had an opportunity to observe each other's lesson, and both teachers noted how valuable this was. However, the nature of their learning differed. Miss Han commented on the lesson (and other observed lessons on the same topic) critically and developed a new design of the lesson based on these reflections.

Miss Han gave positive comments on the lesson taught by Mr Wu with respect to a clear instructional objective and reasonable procedures but criticized the appropriateness of the manipulative activity. In an interview, she stated:

> Instructional objectives are clearly identified, the overall structure of instructional design is delicate, the phases of instruction design are reasonable: from simple to complex. One highlight is emphasizing the methods of discovering patterns from quantitative and geometrical perspectives; I feel this is very good. During the entire process, the teacher engaged students to think along a clear pathway. However, the exploratory activity was not implemented appropriately; I suggest to revise this phase; in addition, the time for students' activities was not enough, although he asked two students to present on the board, but I asked other students, they were not clear.

In addition, since she observed another teacher teaching the same topic, she had an opportunity to compare the three lessons. Drawing on comparisons between the lessons, she redesigned her lesson and wrote a reflection describing her experience. She believed that "in an activity-based lesson, if students are able to engage, find patterns, and master methods of discovering patterns, gain something, it is an effective lesson."(From her redesign and reflection report). The new design and her beliefs about effectiveness of activity-based lessons demonstrated that Miss Han reached a deeper understanding of teaching activity-based lessons that is aligned with the new curriculum.

In his journal reflection, Mr Wu described what he learned from observing Miss Han's lesson. He developed the ideas on how to launch a task, engage students in learning through playing games, and use multiple teaching strategies to differentiate instruction. He further realized that it is critical to create a safe classroom environment that is engaging, encouraging, appreciates students' ideas, and provides mathematically rich tasks. Meanwhile, he realized that "there is a huge difference between the experienced teacher and me regarding instructional language and instructional appearance. So I need to make great effort to improve." (From his reflection report). After observing the public lessons, "I realized the same topic could be taught in totally different styles, effectively. There is no single answer to how to teach an activity-based lesson." Both demonstrating teachers appreciated multiple ways of teaching the same topic and identified rooms from improvement.

Discussion and conclusion

Critical features of teacher learning through PLS

This study demonstrates the mechanisms of teacher learning through PLS from the perspective of expansive learning. There are two dimensions of teachers' expansive learning. Vertically, teacher learning expands from school-based TRGs to district-based LSGs through boundary crossing under brokering by knowledgeable teaching researchers. Horizontally, teachers further expand their learning by comparing and discussing exemplary lessons on the same topic developed by the different PLS groups. Thus teacher learning expands in both dimensions, which creates a networking of professional learning. During the process of teacher learning, two critical features surrounding the research lesson development should be highlighted. The first feature is transformation of the object – the lesson, which through a repeated cycle of designing, teaching, and revising, makes teaching research lessons a deliberate practice (Ericsson *et al.*, 1993). The second feature is the comparison of public research lessons that function as boundary objects for the teachers to work across different communities, which enriches teacher learning by adding more chances of boundary crossing. Teachers learning through parallel lesson study could be summarized in Table I.

Transformation of the object through a repeated cycle. Researchers have argued that teaching public lessons as a deliberate practice can contribute to teacher's expertise development (Han and Paine, 2010; Huang *et al.*, 2011). When the participant teachers in the current study conducted the PLS activities, they repeatedly practiced the core aspects of instruction with immediate feedback from their colleagues and district teaching researchers. Teaching research lessons as a deliberate practice explained the mechanics through which the teachers could improve knowledge and skills for teaching mathematics. Expansive learning frames teacher learning that happened in networks where teachers co-constructed an object – conducting PLS as a collective activity, and implemented the new objects in practice. The activity was not only an individual teacher's learning journey. It was also an expansive learning process during which the participant teachers and the teaching researchers travelled across community boundaries to collaboratively create and transform research lessons.

This study depicts teachers' learning as transformation of objects including instruction objectives and use of mathematics tasks. Due to the unique features of PLS, such as repeated rehearsal teaching and immediately peer and expert inputs, teacher learning was recursive and networking. It was clear that teacher learning happened from "isolated individuals to collective and networks" (Engeström and Sannino, 2010, p. 5) within a school-based TRG, a LSG, and across LSGs. The professional learning activity designed as repeated cycles of rehearsal lessons with immediate feedback transformed the research lessons' focus on knowledge-oriented goals to method-process-oriented goals. PLS supported the teachers in implementing these goals smoothly and coherently.

Studies have revealed when educational reforms pose new innovative ways for instruction, teachers sometimes need to unlearn what they had been doing and believing (Cohen and Hill, 2001). Without sufficient, effective support, the educational reforms would likely fail to result in changes in the classroom. In the current study, the

Major factors influencing teacher learning	Repeated rehearsal lessons	Observation of other teachers' lessons	Peers' observations and feedback	Knowledgeable experts' feedback
Setting appropriate instructional goals	X	X	X	X
Selecting and implementing mathematics tasks	X	X	X	X
Developing professional vision	X	X		X
Developing self-development awareness	X	X		X

Notes: X presents that a type of teacher learning mainly occurs during a specific activity. For example, "developing professional vision" mainly occurs during "repeated rehearsal lessons", "observation of other teachers' lessons", and "expert's feedback"

Table I. Characteristics of teacher learning through parallel lesson study participant teachers received support from their colleagues and teaching researchers to study how to teach activity-based lessons. Representing a significant shift from traditional ways of teaching mathematics, an activity-based lesson is an innovative way to teach and learn mathematics called for by the Ministry of Education in China (Ministry of Education, P. R. China, 2011).

The shift from teaching mathematics in a traditional way to teaching mathematics through problem solving or activities reflects that the teachers had aligned their instructional objectives to the new curriculum aimed to "understand and master basic mathematics knowledge and skills, experience and use mathematics thinking and methods, and obtain basic mathematical activity experience" (Ministry of Education, P. R. China, 2011, p. 42).

During the PLS process, the selection and sequences of mathematics tasks in both research lessons were improved significantly. These findings further support the observations made by other researchers (Han and Paine, 2010; Huang and Li, 2014). As teaching activity-based lessons is a new instructional approach for them, the teachers spent time polishing instructional procedures to produce more coherent learning experiences for the students.

When the teachers and teaching researchers collaborated on creating research lessons, their professional perspectives and expertise were shared, refined, negotiated, and changed across multiple communities. The teaching researchers' expertise in both teaching and research helped the teachers make sense of the innovative teaching philosophy and promoted some specific teaching strategies to be implemented in practice. The teachers developed new views on teaching and learning mathematics through boundary crossing and community building. The participant teachers extrapolated these ideas from others in their own classroom teaching through "taking of the other into account, in light of a reflexive knowledge of one's own perspective" (Boland and Tenkasi, 1995, p. 362; cited from Akkerman and Bakker, 2011, p. 145). According to Clarke and Hollingsworth (2002), teacher professional growth occurs through iterative processes of enactment and reflection between four domains. The external domain represents the systems and policies that stimulate and shape teachers' learning; the personal domain represents teachers' characteristics such as attitudes, beliefs, and knowledge; the domain of practice represents teachers' instructional practice; and the domain of Consequence represents students' learning and other outcomes interpreted by teachers as a consequence of their professional actions. Participation in the repeated cycle of lesson study allows teachers to experience the process of iterative process of enactment and reflection between multiple domains, which enhances teachers' development.

Boundary crossing through comparing public study lessons. The final public exemplary lessons served as insightful "boundary objects" for teachers from both LSGs (communities) to observe and reflect, which "played an important role in the expansion of the shared objects" (Engeströ́m and Sannino, 2010, p. 13). Compared with regular lesson studies, the process of conducting PLS "results in an expanded set of perspectives and thus a new construction of identity that informs future practices" (Akkerman and Bakker, 2011, p. 146). Through comparing their own teaching with others, each teacher developed an awareness of improving their teaching and found directions for improvement. The experienced teacher tried to develop an innovative teaching design by addressing the weaknesses she identified in others, while the novice teacher realized his weaknesses and identified areas to improve. Their increased awareness of strategies for designing and implementing an activity-based lesson and the relationship between design and implementation had a substantial impact on their professional growth. PLS expands the network of teacher professional learning and brings in more learning opportunities for teachers.

Chinese lesson study is similar to Japanese lesson study in terms of the typical cycle of a lesson study (Lewis and Hurd, 2011; Lewis *et al.*, 2009). Yet, Chines PLS has some unique features. First of all, the repeated teaching at least two times is required, which is one of the key features of Chinese lesson study (Huang and Bao, 2006; Yang and Ricks, 2012). Second, Chinese lesson study is more content-focussed and pedagogy-oriented. This means that Chinese lesson study focusses on developing effective strategies for teaching particular content or for teaching a type of lesson (Huang and Li, 2014), rather than focussing on a general goal of learning or teaching. Third, PLS is guided by knowledgeable teaching researchers who are experienced in both teaching and research on teaching. Fourth, PLS includes another component of comparing and contrasting exemplary lessons on the same topic across LSGs, which extends participant teachers' learning opportunities.

Adaptation of teacher learning through PLS

This study describes an approach of mathematics teacher learning through PLS in China. Since teaching is a culture activity (Stigler and Hiebert, 1999), teacher learning is eventually embedded in the cultural and educational system. To consider whether the Chinese practice could be adapted to other education settings, it is important to identify the underlying factors making it work in China. The cyclic process of research lesson development reveals that there is a "culture of teaching as public activity" in China (Stigler *et al.*, 2013). In China, observing others' teaching, being observed by others, and conducting public lesson are routine parts of teaching practice. Thus, teachers are willing to share their thoughts related to the teaching rather than to the teachers. Second, the knowledgeable experts play critical roles in facilitating teacher learning. The promotion system defines teacher professional expertise at different levels and provides incentives and a mechanism for teachers to develop from junior levels to senior levels (Huang *et al.*, 2010). Mentoring novice teachers is the responsibility of senior teachers. Due to a cultural tradition of respecting seniors in China, mentors and mentees usually develop a harmonic professional and respectful relationship. Third, the ubiquitous teaching research activities provide teachers' learning opportunities to meet their needs (Huang *et al.*, 2010). Therefore we believe that the culture of teaching as public activity, respecting senior teachers, the professional promotion system, and the teaching research system are essential factors that support conducting parallel lesson study.

Conclusion

This study explored how practicing teachers could develop their professional competence through PLS. The expansive learning theory provides a useful tool to capture teacher learning through PLS in China. The participating teachers developed knowledge and skills for teaching activity-based lessons effectively, developed new perspectives of teaching and learning mathematics, and realized the ways to improve their teaching practice. In the context of Chinese education system, transformation of the object (exemplary lesson) and boundary crossing are two powerful lenses to depict teacher learning through parallel lesson study.

The importance of repeated teaching; knowledgeable expert input and boundary objects are featured in this study. Theoretically, this study provides an example of how expansive learning theory could be used to analyze teacher learning through parallel lesson study. Practically, this study investigated a case of how a lesson study approach worked in China, which will provide a catalyst for educators from other education systems to reflect upon what could be learned from this specific Chinese practice.

Note

1. Such as finding equations using four basic operations of addition, subtraction, multiplication and division.

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Appendix. Description of the final research lessons

Miss Han delivered the final public lesson to 53 seventh graders in an auditorium with more than 200 middle school mathematics teachers observing. The lesson lasted 45 minutes, with four major phases: introduction, self-exploratory tasks, application with feedback, and summary.

The lesson was carried out surrounding the four activities:

Activity 1: making observation of the patterns among numbers in a row and a column in a calendar.

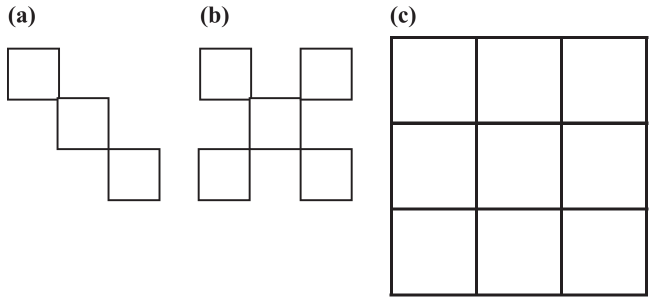
Activity 2: playing a game that students added three consecutive numbers in a row and reported the sum to the teacher who would tell the students what their three numbers were.

Activity 3: exploring various patterns with specific grids in a calendar as shown in Figure A1.

Activity 4: making and justifying various patterns in an arrangement that all odd number are displayed seven numbers in a row.

In a similar environment Mr Wu presented his public lesson of the same topic. The lesson began with an observation of a monthly calendar. The teacher asked students to fill out numbers in a row, a column, and a diagonal diagram (two or three consecutive numbers) when one number was given (See Figure A2. Moreover, students are encouraged to express these patterns in verbal and mathematical language (e.g. in the left grid, the number increases by 1, the three numbers are 17-2, 17-1, and 17).

After finding the patterns, the students are encouraged to express these patterns algebraically, and make conjectures about the relationship between the sum of the three numbers and the number in the middle in a row, and prove the conjecture using algebraic expressions (Task 2). Next the students explored the patterns among numbers in 3×3 grid frames in a monthly calendar (Task 3). The whole class explored two concrete examples to reveal a general pattern about the relationships between the sum of these nine numbers and the number in the center. Finally, the students were guided to develop an algebraic justification for this pattern. The class ends with a summary of the major ideas of the lesson including how to observe the patterns in a calendar, what the benefits using letters to represent numbers, how to effectively present numbers using letters, the connection between mathematics and daily life situation. Moreover, some variation problems were assigned as homework, such as exploring patterns in a "+" grids or 4×4 grid in a monthly calendar.



Notes: (a) Diagonal model; (b) plum-flower model; (c) 3 × 3 grid

Figure A1. Selected grids for student exploration

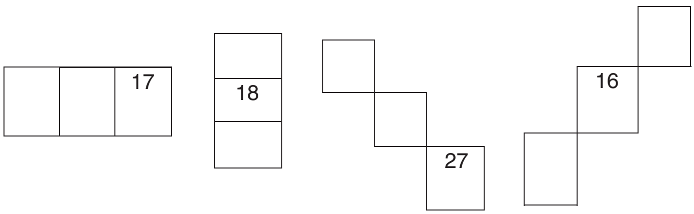


Figure A2. Finding the missing numbers in the given grids in a monthly calendar

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